III. *On the corrections in the elements of* DELAMBRE'S *Solar Tables required by the observations made at the Royal Observatory, Greenwich. By* GEORGE BIDDELL AIRY, *Esq. M.A., Fellow of Trinity College Cambridge, and Lucasian Professor of Mathematics in the University of Cambridge. Communicated by* JOHN FREDERICK WILLIAM HERSCHEL, *Esq. V.P.R.S.*

Read December 6, 1827.

THE attention of the Board of Longitude having been directed to the state of the Solar Tables used in the construction of the Nautical Almanac, the Astronomer Royal was requested to furnish the Board with a comparison of the computed and observed Right Ascensions of the Sun since the erection of the new transit instrument at Greenwich: and I was desired to examine the discrepancies with a view to the discovery of the errors in the elements of the tables. The papers containing this comparison I received at the beginning of June last; and in the last summer, as soon as other engagements permitted, I undertook the laborious work of examining the discordances. The corrections of the elements determined by these calculations agree in general with those that have been obtained from other observations; but in one particular, and that an important one, there is a very remarkable difference. The results of such an inquiry will perhaps be acceptable to the Royal Society; and the singularity of the conclusion which I have mentioned, will make it necessary for me to describe the manner in which I have obtained it.

The number of observations from which this comparison is made is 1212, commencing on July 30, 1816, and terminating on December 30, 1826. The only interruption of any importance is one of about three months, from February 4, to May 22, 1825. These do not include all that are given in the Greenwich Observations, but only those which are likely to have been least affected by irregularities of the clock, &c. in consequence of the observation of standard stars at no great interval. As far as the end of 1820 the observations are reduced by Dr. MASKELYNE's catalogue; after that time they are reduced by Mr. POND's catalogue of 1820, in which the right ascensions of the stars are increased $0'',31$, of time. Though there is sometimes a disagreement in the errors for consecutive days amounting to three or four tenths of a second of time, (undoubtedly produced partly by the errors of observation and partly by those in the calculation of the Nautical Almanac,) yet when the skill of the observers and the excellence of the instrument are considered, I believe it will be allowed that this mass of observations is equal to any that has been used for the purpose of correcting the tables.

A slight inspection of these errors was sufficient to indicate that a correction of the epochs of the sun's longitude and of the longitude of the perigee, with perhaps an alteration of the equation of the centre, would bring the calculated place for any one year sufficiently near to the observed place; and with these corrections only I commenced my calculations. But, upon comparing the discrepancies for different years, I found that there was certainly some other source of irregularity; and such could be found only in the erroneous mass assigned to some of the planets. The masses of Venus and Mars being the only ones which produce any sensible effect on the Earth's motion, and which can be measured in no other way, I supposed them subject to error, and with these five assumed corrections the greatest part of the calculations has been made. A more critical examination showed that there was an error in the assigned mass of the moon; but the rapidity of variation of the lunar inequality allowed me to determine this correction by a more simple process.

Suppose now O to be the observed error of the tables, the sun's observed place being reduced by Dr. MASKELYNE's catalogue; e the error of MASKELYNE's place of the equinox in Æ, or the quantity by which the right ascensions of all the stars in his catalogue ought to be increased, both expressed in seconds of time. Then $O-e$ is the real error of the tables in Æ. After January 1st, 1821, when the observations are reduced by Mr. POND's catalogue, in which the right ascensions are increased by ,31 of time, the error of the tables is $O+,31-e$. The corresponding errors in longitude, expressed in seconds of space, are $15 \sec 23^\circ 28' \times \cos^2 \text{dec} \times \overline{O-e}$; and $15 \sec 23^\circ 28' \times \cos^2 \text{dec} \times \overline{O+,31-e}$. Now if the proper corrections were applied to the elements of the tables, the sum of the quantity just found, and the alteration produced in the longitude by those corrections, would, if the observations were accurate, be nothing; and if there be no constant cause of error in the observations, upon adding a great number of quantities thus formed, the errors may be expected nearly to destroy each other, and the sum may be assumed equal to nothing. Let X be the number of seconds of space by which the epoch of the sun's longitude ought to be increased; Y the increase in seconds of the epoch of the longitude of perigee: Z the increase which must be made in the greatest equation of the centre: suppose the mass of Venus to be increased in the ratio of $1 : 1 + V$, and that of Mars in the ratio of $1 : 1 + M$. Then $X - Y$ is the increase of the mean anomaly. Now the tables contain the alteration in the equation of the centre for an increase of $10'$ in the mean anomaly, and for a diminution of $17'',177$ in the greatest equation of the centre. The sun's true longitude then being taken from the Nautical Almanac, and his true anomaly found by subtracting the longitude of the perigee, the mean anomaly corresponding was found in the tables, and the alterations of the equation of the centre for $+ 10'$ in mean anomaly and $- 17'',177$ in the greatest equation were taken out: call them a and b. By the alteration of epochs and of the equation of the centre, the tabular longitude would be increased by $X + \frac{a \times (X-Y)}{600} - \frac{b Z}{17,177}$. The perturbations of Venus and Mars are increased each by a constant to make them always positive; (Delambre has not mentioned the values of the constants, but they appear to be respectively $16''6$, and $6'',5$; they are subtracted from the equation of the centre): call these c and d, and let f and g be the tabular perturbations. Then the real perturbations are $f - c$ and $g - d$; and by increasing the masses of Venus and Mars, the sun's tabular longitude would be increased by $V(f - c)$ and $M(g - d)$. If then we neglect for the present the lunar equation, which we may do, since every group of equations will comprehend several lunations, we shall have a series of equations similar to the following, each of which is true excepting the errors of observation;

$$0 = 15 \sec 23^\circ 28' \times \cos^2 \text{decl} \times \overline{O - e} + X + \frac{a(X-Y)}{600} - \frac{bZ}{17,177} + V(f - c) + M(g - d).$$

Dividing this by 15 sec 23° 28′, and putting $\frac{X}{15 \sec 23^\circ 28'} = x$, $\frac{X - Y}{600 \times 15 \times \sec 23^\circ 28'}$

$= y, \frac{-Z}{17{,}177 \times 15 \times \sec 23^\circ 28'} = z, \frac{V}{15 \sec 23^\circ 28'} = v, \frac{M}{15 \sec 23^\circ 28'} = m,$ the equation becomes

$$0 = \cos^2 \text{decl} \times \overline{O - e} + x + ay + bz + v(f - c) + m(g - d).$$

The equations thus formed were to be divided into groups of two kinds: one kind in which the coefficient of one of the unknown quantities was large, another in which it was small or negative: and by subtracting the sum of one group from the sum of the other, an equation would be obtained in which the coefficient of that quantity would certainly be large, and in which the other coefficients probably would not be large. As there was some uncertainty respecting the values of c and d, it appeared desirable to eliminate them from all the equations but one: this was done by dividing the equations into groups of equal numbers and subtracting one from the other. Thus the equations were divided into eleven groups containing 606 equations, in which the value of a was positive or $> - 3{,}8$; and eleven groups containing 606 equations, in which a was negative and $< - 3{,}8$; their difference gave this equation:

$$(\beta)\ldots 0 = + 85{,}484 - e \times 9{,}382 + y \times 15048{,}9 - z \times 368{,}58 + v \times 1026{,}7 + m \times 598{,}4.$$

They were again divided into eleven groups containing 606 equations, in which the value of b was positive and $> 2{,}5$; and eleven groups containing 606 equations, in which b was $< 2{,}5$ or negative: their difference gave

$$(\gamma)\ldots 0 = - 22{,}785 - e \times 13{,}832 - y \times 174{,}5 + z \times 13319{,}28 + v \times 517{,}3 + m \times 636{,}0.$$

They were then divided into thirteen groups of 606 equations, in which the value of f exceeded 15,4; and thirteen groups of 606 equations, in which f was less than 15,4: their difference gave

$$(\delta)\ldots 0 = + 61{,}465 - e \times 1{,}536 + y \times 2905{,}7 + z \times 95{,}50 + v \times 6655{,}1 + m \times 223{,}2.$$

Similarly, they were divided into nine groups of 606 equations, in which g exceeded 6,3; and nine groups in which g was smaller: from their difference

$$(\varepsilon)\ldots 0 = + 65{,}860 + e \times 6{,}678 + y \times 5706{,}9 + z \times 3944{,}46 + v \times 439{,}9 + m \times 2307{,}2.$$

Lastly, by adding together all the equations,

$$(\alpha)\ldots 0 = - 177{,}278 - e \times 1118{,}282 + (x - vc - md) \times 1212 - y \times 1744{,}3 + z \times 1418{,}96 + v \times 19319{,}1 + m \times 7519{,}8.$$

As it was probable that the epoch of the sun's longitude ought to be altered, it was to be expected that the secular motion of the tables was incorrect. I had supposed, however, that this error might be neglected in the investigations, and that the results would be sufficiently accurate for the mean time of the observations: but an examination of the groups showed that this was not the case. The equations in which the coefficients of x and those of y were alternately positive and negative, as well as those in which the coefficients of v were alternately great and small, were distributed with tolerable uniformity over the several years. But it was not so with those in which the coefficients of m were alternately large and small. A single group in which the coefficients of m were large, extended over nearly two years at the beginning of the period of the observations, and comprehended 219 observations. The equation then (ε) intended to have the coefficient of m large, would be of the same nature as if the sum of the equations for several years were subtracted from the sum of the equations for several years previous; and would therefore be much affected by the error in the secular motion. This was taken into account in the following manner: let S be the number of seconds of space by which the secular motion ought to be increased; and let X now represent the correction of the epoch for 1816. Then in q years after 1816 the tabular longitude would have been greater, had the secular motion been correct, by $\frac{q\mathrm{S}}{100}$; and in r years by $\frac{r\mathrm{S}}{100}$. If in this interval N observations had been made, distributed almost uniformly over the interval, the sum of all the increments of the tabular longitude would have been nearly $\frac{\mathrm{N}}{2}\left(\frac{q\mathrm{S}}{100}+\frac{r\mathrm{S}}{100}\right)=\frac{\mathrm{S}}{200}\cdot\mathrm{N}\,(q+r)$ in seconds of space. By this quantity the errors of the tables (in the original equations containing X, Y, &c.) ought to be increased: and therefore, in the equations containing x, y, &c. they ought to be increased by $\frac{\mathrm{S}}{3000\,.\,\sec 23^\circ\,28'}\times\mathrm{N}\,(q+r)$. Let $\frac{\mathrm{S}}{3000\,.\,\sec 23^\circ\,28'}=p$: then the first term of each group, (in which N is the number of equations, and q and r the years and fractions of years from the beginning of 1816 to the first and last observation respectively, of that group,) must be increased by $p\times\mathrm{N}\,(q+r)$. Applying this to all the groups, it is found that (α) ought to be increased by $+14051\,.\,p$; (β) by $-220\,.\,p$; (γ) by $+393\,.\,p$; (δ) by $-241\,.\,p$; and (ε) by $-3817\,.\,p$.

Solving the equations thus augmented, we find

$$
\begin{aligned}
x - vc - md &= ,37718 + e \times ,9696 - p \times 24,182 \\
y &= -,0043027 + e \times ,000964 - p \times ,0697 \\
z &= ,0029488 + e \times ,001419 - p \times ,1278 \\
v &= -,006674 + e \times ,000049 - p \times ,0004 \\
m &= -,021682 - e \times ,00775 + p \times 2,042
\end{aligned}
$$

And (supposing $c = 16,6$, $d = 6,5$), $x = ,12547 + e \times ,9200 - p \times 10,915$; whence

$$
\begin{aligned}
X &= +\ 2,0518 + e \times 15,045 - S \times ,05458 \\
Y &= +\ 44,2690 + e \times 5,589 + S \times ,1545 \\
Z &= -\ ,8283 - e \times ,399 + S \times ,01096 \\
V &= -\ ,1091 + e \times ,0008 - S \times ,000002 \\
M &= -\ ,3546 - e \times ,1267 + S \times ,01021
\end{aligned}
$$

in which X, Y, Z, and S are expressed in seconds of space, and e in seconds of time.

The Astronomer Royal, in his latest catalogue, has made $e = 0'',20$. To determine S we have the following data. From the researches of M. BURCKHARDT (Conn. des Tems, 1816,) it appears that the correction of the epoch in 1783, using MASKELYNE's catalogue, was $0'',25$: in 1801, $0'',8$. And from the expression above for X it appears that the correction, using MASKELYNE's catalogue, was + 2,05 in 1816 + 5,458 years, or in the middle of 1821. The comparison of this with the correction for 1783 gives $S = 4'',7$: the comparison with that for 1801 gives $S = 6'',1$. Now if there be in the sun's motion any inequality of long period, the value of S which is wanted here is not the real increase of mean secular motion, but that which, independent of the inequalities taken into account in the tables, but including all others, applies to the period of the observations. I have therefore taken $S = 6'',0$. After these substitutions I find that

The epoch for 1816 ought to be increased by $4'',734$; or, more exactly, free from any uncertainty respecting the value of S, the epoch for 1821,5 ought to be increased by $5'',061$.

The epoch of the perigee ought to be increased by $46'',3$.

(These epochs are to be measured from the equinoxial point adopted by Mr. POND in his catalogue of 1826.)

The greatest equation of the centre ought to be diminished by $0'',84$.

The mass of Venus ought to be reduced in the proportion of 10000 : 8911, or 9 : 8 nearly.

The mass of Mars ought to be reduced in the proportion of 10000 : 6813, or 22 : 15 nearly.

Hitherto I have not considered the possible error in the coefficient of the lunar equation. The variations of this equation are so much more rapid than any of the others, that the other corrections may be determined independently of it, and it can be determined independently of them, and even without reducing the errors to errors of longitude. I have merely arranged the observed errors in Æ from the middle of 1816 to the end of 1820 in two groups, one comprehending all the observations between new moon and full, and the other all the observations between full moon and new; from each group I have found a mean, and have taken the difference of the means. The same has been done from the beginning of 1821 to the end of 1826. Thus, to the end of 1820,

Mean of 248 errors between new moon and full = – ,0825.
Mean of 265 errors between full moon and new = – ,1609.
Excess of the former + ,0784 in seconds of time.

After 1820.

Mean of 374 errors between new moon and full = –, 4493.
Mean of 325 errors between full moon and new = – ,5445.
Excess of the former + ,0952.

Mean of the whole ,0881 in seconds of time, equivalent to $1'',322$ of space.

To find the alteration which this requires in the coefficient of the lunar equation, suppose k to be that alteration: then the correction to the sun's longitude would be very nearly $k \times$ sin diff. long. of sun and moon. Now in the number of observations that we have taken, we may suppose that the difference of longitudes of the sun and moon has had different values between 0° and 360° without any remarkable preponderance of any particular values. To reduce this to calculation, suppose while the angle increased from o to π,

n observations were made at the intervals $\frac{\pi}{n}$: then we must find the mean of the quantities $k\sin\frac{\pi}{n}$, $k\sin\frac{2\pi}{n}$, &c. up to $k\sin\pi$: in other words we must find the value of $\frac{k}{n}\Sigma\sin\frac{x\pi}{n}$. Now $\Sigma\sin\frac{x\pi}{n}=-\frac{\cos\overline{x-\frac{1}{2}}\cdot\frac{\pi}{n}}{2\sin\frac{\pi}{2n}}$: which from $x=1$ to $x=n+1$ is $\frac{2\cos\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}}=\frac{1}{\tan\frac{\pi}{2n}}$: and the mean $=\frac{2k}{2n\tan\frac{\pi}{2n}}$, which when n is very great becomes nearly $=\frac{2k}{\pi}$. This then is the mean quantity by which the longitudes between new moon and full, so far as they depend on this equation, are too great, and similarly $\frac{2k}{\pi}$ is the mean quantity by which the longitudes between full moon and new are too small. Taking the difference then, $\frac{4k}{\pi}=$ difference of mean errors $=1'',322$; whence $k=1'',04$. And since the sun's longitude, as far as it depends on this, is too great between new moon and full, at which time the lunar equation increases the longitude, it follows that the coefficient of the lunar equation ought to be diminished by $1'',04$. The coefficient in DELAMBRE's tables is $7'',5$: hence, if the moon's parallax be not altered, the quotient of the moon's mass by the moon's mass + the earth's mass is to be diminished in the ratio of 29 : 25 nearly.

If these deductions could be relied on, we should have

Mass of Venus $=\frac{1}{401211}\times$ that of the Sun.

Mass of Mars $=\frac{1}{3734602}\times$ that of the Sun.

Mass of the Moon $=\frac{1}{80,4}\times$ that of the Earth.

And the limits of the errors of DELAMBRE's tables, roughly estimated, would be as follows,

Error in epoch for 1830		$-5'',6$
Greatest error from error in place of perigee . . .	$\pm 1'',5$	
Greatest error from error in greatest equation of centre	$\pm 0,8$	
Greatest error from the combination of these		$\pm 1,7$

Greatest error from error in mass of Venus	$\pm$ 1″,5
Greatest error from error in mass of Mars	$\pm$ 1 ,9
Greatest error from error in mass of the Moon	$\pm$ 1 ,0
Greatest possible negative error	— 11″,7
Greatest possible positive error	+ 0″,5

I shall now compare these results with those which have been found from an examination of some of Dr. MASKELYNE's observations.

In the Connaissance des Tems for 1816, M. BURCKHARDT has given the results of a comparison of DELAMBRE's tables with nearly four thousand of MASKELYNE's observations, extending from 1774 to 1810. The following are his most important conclusions.

1st. The correction of the epoch in 1801 was + 0″,8. In 1783, + 0″,25. In 1752, — 3″,0. The latter was found from 310 of BRADLEY's observations, and the result seems doubtful, from the uncertainty of the reductions.

2nd. The correction of the longitude of the perigee in 1783 was + 25″; in 1801 it was — 2″,7. If these places of the perigee be used, the variable part of aberration is not to be applied, in order to find the sun's apparent place.

3rd. The correction of the greatest equation of the centre in 1783 was — 0″,84; and in 1801, — 1″,23.

4th. The coefficient of the lunar equation to be diminished from 7″,5 to 6″,8.

5th. The mass of Venus to be diminished $\frac{1}{9}$th.

6th. The mass of Mars to be diminished $\frac{1}{20}$th: which produces an almost insensible effect on the sun's longitude.

The general agreement of my conclusions with those of M. BURCKHARDT is highly satisfactory. The correction of the equation of the centre and the diminution of the mass of Venus are absolutely the same: the diminution of the coefficient of the lunar equation differs very little. In the diminution of the mass of Mars there is a sensible difference: and though my equation for m is not so favourable for its exact determination as those for y, z, and v, yet I am inclined to think that M. BURCKHARDT's diminution is not sufficient. The slight disagreement in the increase of secular motion deduced from the comparison of my correction of the epoch with the two given by M. BURCKHARDT, (a correction which in general is liable to less uncertainty than any other,) seems to show that there is still some very small inequality in the sun's motion not included in the tables.

But our deductions as to the correction of the place of the perigee do not present the same agreement. It must be observed that the variable part of aberration is included in the longitude given by M. BURCKHARDT's corrected perigee. Now the effect of this variable part is the same as if the longitude of the perigee were increased by 10″,1. Consequently M. BURCKHARDT's correction of the perigee ought to be diminished by 10″,1. Thus we have,

Correction of perigee in 1783, + 14″,9
Correction of perigee in 1801, − 12″,8
Correction of perigee in 1821, + 46″,3

The motion of the perigee then appears to be of the most irregular kind. Of the accuracy of the correction for 1816, as established by Mr. POND's observations, there can be no doubt. Independently of the very great care which has been used, by systematic checks on every part of the operations, to insure accuracy in the numerical calculations, it is sufficient to glance at the discrepancies of the observed and calculated Æ, in order to see that the longitude of the perigee must be increased. The negative errors of the tables are invariably greatest in summer. The necessity of diminishing the masses of Venus and Mars, and even of diminishing the equation of the centre, is not evident till the equations are formed: but the error and the kind of error in the place of the perigee will never be doubted by any one who has seen the observations. The equation also (β) in which the coefficient of y is large, is very favourable for its exact determination.

I can see only two ways in which this singular irregularity can be accounted for. One is by supposing that the term in the motion of the perigee which depends on the square of the time is incorrectly calculated. I have too much confidence in the accuracy of the results in the Mécanique Celeste to suppose there the existence of an error sufficiently great. The other is by supposing some yet undiscovered inequality of the form $a \cdot \sin(b\,\theta + c)$ where θ is the sun's mean longitude and b a coefficient differing very little from unity. This I suspect to be the true cause of the discordance of theory and observation.

The corrections which I have stated as the result of this examination differ in some degree from those deduced in the Phil. Trans. for 1827, p. 65, &c. from Mr. South's observations. The smallness of the number of observations there used, and the rejection of any alteration of the planetary perturbations, are sufficient to account for this difference.

G. B. Airy.

Trinity College, Cambridge,
Oct. 3rd, 1827.

POSTSCRIPT.

I have the satisfaction of stating to the Royal Society, that since the communication of the paper above, my conjecture with regard to the origin of one of the irregularities noticed in it has been completely verified. Upon examination of the planetary theory, I find that in consequence of the action of Venus, the Earth's motion in longitude is affected with an inequality for which the expression, taking the mass of Venus as determined in this paper, is

$$2'',6 \times \sin\left\{8 \times \text{mean long. Venus} - 13 \times \text{mean long. Earth} + 39^\circ\ 57'\right\}.$$

The period of this inequality is about 240 years. This term accounts completely for the difference in the secular motions given by the comparison of the epochs of 1783 and 1821, and by that of the epochs of 1801 and 1821. From the known relations of terms in the investigations of physical astronomy, it will be seen that there must be in the expression for the Earth's longitude, terms, probably sensible, of the forms

$$A \cdot \sin\left\{8 \times \text{mean long. Venus} - 12 \times \text{mean long. Earth} + B\right\},$$

and

$$C \cdot \sin\left\{14 \times \text{mean long. Earth} - 8 \times \text{mean long. Venus} + D\right\};$$

and these may account for the alteration of the equation of the centre, and the irregular motion of the perigee. The earth's latitude also must be affected with similar terms. I have been prevented from calculating these terms, and from extending the former so as to include the parts depending on the secular variations of the elements, and even from examining some parts so carefully as I could wish, by the excessive labour attending these investigations. The term calculated is of the 5th order; and I believe it may be fairly stated that the labour of the calculation is twenty times as great as for the long inequality of Saturn, and far greater than for any term hitherto treated of. I shall resume the investigations as soon as I have sufficient leisure and spirits: and I propose then to lay before the Society a more detailed account of the calculation.

G. B. AIRY.

Trinity College,
December 16*th*, 1827.

Erratum in the Paper on Mr. SOUTH'S *observations.*

In the Phil. Trans. for 1827, page 69, line 16,—*for* $1^s\ 30°$, $4^s\ 30°$, $7^s\ 30°$, $10^s\ 30°$,—*read* $1^s\ 15°$, $4^s\ 15°$, $7^s\ 15°$, $10^s\ 15°$. This error has not affected the calculations.